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RESEARCH PAPER

Probabilistic Prediction of Failure in Columns of a Steel Structure Under Progressive Collapse Using Response Surface and Artificial Neural Network Methods

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Abstract

Much attention has recently been paid to the issue of progressive collapse, which is associated with the uncertainties that may affect the accurate assessment of the safety of the structures. Probabilistic analysis can be used to quantify the probabilistic safety of structures under extreme loadings. Since the columns play a key role in the stability of the structures subjected to the progressive collapse and they are very prone to failure, this research focuses on estimation of the failure probability in these structural elements. Monte Carlo simulation is used to perform the probabilistic analysis in a steel structure. The ratio of the axial force demand to the inelastic buckling capacity in columns adjacent to the damaged column is considered as the implicit limit state function. Artificial neural network and response surface methods are used to estimate an explicit function to save computational time. The results obtained from this study can be used to rehabilitate damaged structures using the effective role of each random variable on the structural responses which have been determined by the sensitivity analysis.

Keywords Progressive collapse · Failure probability · Response surface method · Artificial neural network · Sensitivity analysis

1 Introduction

Progressive collapse is the local damage to the structure so leads to partial or general failure due to reduction in stiffness of the structural members. Damage can be caused by unusual loads such as gas explosion, vehicle collision, fire and human error in the design and construction of structures (Ellingwood and Dusenberry 2005).

Extensive research has been conducted on the progressive collapse of the structures in recent years. One of the most important goals of the progressive collapse analysis is the evaluation of the vulnerability of the structural systems to locate the damage (Gerasimidis 2014). In many studies, an alternative load path (ALP) method proposed by design regulations (Pioldi et al. 2017; Xia and Brownjohn 2004) is used for assessment the collapse behavior of the structures. In the APM, the ability of the remaining structures is investigated to transfer gravity loads to the ground by removing a vertical load-bearing structural member. Fu (2013) examined the collapse of building structures due to creation of high shear forces resulting from the removal of columns using the APM. Nica et al. (2018) carried out a numerical analysis on progressive collapse of the irregular structures due to the blast. They considered a demolition scenario in columns to illustrate the effective operation of the applied element method in the redistribution of the load after the failure in the columns. Their result showed that the structures were sufficiently resistant to the progressive failure due to seismic design. Tavakoli and Alashti (2013) evaluated the potential of the progressive collapse in the multi-story moment resisting steel frame buildings subjected to the lateral loading. Tavakoli and Kiakojouri (2014) proposed several methods to prevent the spread of damage using the concept of robustness. Also, they (2015) evaluated the role of initial failure location and the number of floors for the potential of the progressive collapse in the structures. Tavakoli and Moradi (2014) assessed the potential of the progressive failure in retrofitted structures which are damaged before. Tavakoli

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et al. (2015) conducted a progressive collapse analysis on the structural frames to evaluate resistance of the structure under seismic progressive failure. They adopted a method that localized failures and prevented the spread of damage to the intact spans. Tavakoli and Hasani (2017) assessed the effects of different characteristics of the seismic excitations on the potential of the seismic progressive collapse in the steel special moment resisting frame structures. Tavakoli and Moradi (2018) investigated the potential of the structures against the progressive failure using a robustness index for structures with different lateral load-bearing systems. They proposed a simple energy-based method for conducting robustness analysis. Recently, Biagi et al. (2020) proposed a simplified method for the assessment of the frame structures subjected to sudden column removal, accounting the dynamic effects that raise. This can provide an interesting insight into the methods. Karimiyan (2020) proposed a collapse distribution pattern for the structural elements under the removal of the internal column to predict the progress of progressive collapse caused by seismic loads. Mehdizadeh et al. (2020) assessed the sidesway collapse capacity of steel moment-resisting frames under seismic loads. The results of their study showed that special steel moment-resisting frames have a larger collapse capacity than intermediate and ordinary moment-resisting frames. In the assessment carried out by Agarwal and Varma (2014), the effective role of the columns was identified in maintaining the stability of the structure under the fire event. The first buckling that happens in damaged columns can be checked to control the stability of the damaged structure (Pantidis and Gerasimidis 2017). Gerasimidis et al. (2015) investigated collapse mechanisms due to loss of stability in the columns under progressive failure. The results of their evaluation showed that two adjacent columns buckled at the level of the load less than the design load with the removal of a corner column in the structure. After failure of these two columns, the buckling instantly occurred in the next two adjacent columns. Finally, the entire structure collapsed without making significant plastics in the structural system after the collapse of the five columns. Their work showed that there is a strong relationship between the loss of stability in structural system and the phenomenon of the progressive failure. Jiang and Chen (2012) proposed a method for evaluating the safety of the structures that can be identified by key element in the structure under the progressive collapse. They found that this method can predict the structural vulnerability.

In previous studies for assessment of the progressive collapse in the structures, the behavior of the structural collapse modes such as instability modes of the columns adjacent to the damaged columns was investigated in the form of the deterministic analysis. Many engineering problems have different uncertainties that may affect the potential of the failures in the structures. The most important uncertainties are commonly related to the loads and materials. The probability of the failure in the structures under these uncertainties can be determined by the reliability analysis. Using the reliability analysis can also quantify the safety of each member of the structure under damage imposed to the structure. Then, it can be decided whether a structural member is to be repaired or replaced (Santosh et al. 2006). In recent years, much attention has been devoted to the probabilistic assessment of the collapse in the structures (Li et al. 2016; Yu et al. 2016). For instance, Felipe et al. (2018) proposed a reliability-based approach that can determine the key element in the structures under the progressive collapse. So that, this recognition can be used for an optimal design with the aim of a highly robust structural design. Abdollahzadeh and Faghihmaleki (2018) proposed a method for assessing the probabilistic risk of the reinforced concrete buildings subjected to the two hazards as blast and earthquake loads. Izzuddin et al. (2012) assessed the probabilistic risk of the steel structures under events that lead to progressive collapse. They consider uncertainties in the hazards that lead to local damage in the structure. They used the first-order reliability method (FORM) for estimation of the reliability index in the structures. Since this method may be associated with a large number of errors and does not take into account nonlinear terms, Chen et al. (2016) suggested a new approach to assess the reliability of the steel structures subjected to the progressive collapse known advanced FORM that solve the limitations of the FORM method. They also proposed an analytical model for beam which considers the variation of the internal energy in the beam above the removed column. Finally, they presented a method for calculating a robustness index based on the acceptable probability of failure for structures under the progressive collapse. Moradi et al. (2019) conducted a probabilistic assessment on the collapse time of a steel structure under fire event. They also estimated the probability of failure of an intact structure and a previously damaged structure for a specified failure time under fire. Moradi et al. (2020) also conducted a sensitivity analysis on the failure time of the concrete structures under post-earthquake fire. Javidan et al. (2018) examined the probabilistic responses of the steel structures subjected to the collision of vehicles. They used the neural network methodology along with integration-Gauss technique for performing the reliability analysis. Their results showed that the structure is more vulnerable when the corner column is subjected to the collision of vehicles. They also concluded that the probability of exceeding different damage states is greater in the weak axis of the structure under removal of the column.

Various methods such as artificial neural networks (ANN) and response surface method (RSM) can be used to obtain the probability of the collapse of the structures which is required to carry out a large number of finite element analysis. Therefore, these methods can be used instead of a large number of traditional finite element analysis to save the computational time. When the nonlinearity in the structural model is very high, the neural network can estimate the structural responses with less error using the activation functions (Javidan et al. 2018) compared to the functions used in the response surface method. The probabilistic safety of a structural system is evaluated by considering two components including the applied load and the strength of each member. To determine the behavior of a structure, an explicit equation is required based on random variables that are characterized by their probability distribution. Safety assessment method with implicit functions can be simplified to obtain an explicit limit state function. This function can be determined by fitting a surface using the response surface method. This method significantly reduces the time of calculation of the structural responses (Freudenthal et al. 1966). An artificial neural network (ANN) is also very effective in the engineering research for the estimation of the functions and prediction of the structural behavior in probabilistic evaluation (Šipoš et al. 2013). In ANN, a multilayer feed forward approach can be utilized to estimate the implicit function using the actual structural responses obtained from the finite element analysis methods (Deng et al. 2005). The accuracy obtained to estimate the functions in these methods is very dependent on the sampling method. The method of Latin hypercube sampling can be used to reduce the number of samples which is based on the reduction in variance (Ditlevsen and Madsen 1996).

The main objective of this study is probabilistic assessment of a steel structure with considering the uncertainty parameters for buckling modes appeared in the columns adjacent to the damaged column due to progressive collapse. Assessment of collapse under abnormal conditions is still a controversial issue, especially when a reliability analysis is performed. Structural analysis under these uncertainty conditions may lead to numerical instability or spend a lot of time for convergence of analysis. In the present research, collapse analysis of the building structures is performed based on the probabilities analysis using Monte Carlo simulation. At first, Monte Carlo analysis is performed based on explicit equations derived from the response surface method and the responses obtained from training the neural network. Then, the most effective uncertainty variable on the structure response is determined using sensitivity analysis. A steel special moment frame is used under extreme loads to illustrate the application of the methods mentioned for probabilistic analysis, which has been considered uncertainties in loadings and features of its materials.

2 Probabilistic Analysis Methods with Implicit Functions

A limit state function may not be an algebraic function based on the random variables, and it may be composed of response quantities obtained from a finite element model. These functions are called implicit limit state functions. When a specific function of random variables is not available, various methods can be used to estimate the explicit function. The response surface method is one of the methods to approximate a close-form expression for the limit state function. Then, Monte Carlo simulation can be performed using the obtained approximation function, which drastically reduces the computational time compared to the case where the limit state functions are implicit (Haldar and Mahadevan 2000). Also, Monte Carlo simulation based on neural network method is another method of probabilistic analysis to deal with implicit functions (Deng et al. 2005).

2.1 Response Surface Method (RSM)

In some probabilistic problems, the limit state function g (x) cannot be explicitly expressed in terms of the random variables X. Response surface methodology (RSM) is used to save the cost of computing. This method is used to convert an implicit function to an explicit function. Then, the Monte Carlo simulation can be used to predict the probability of the failure with the new limit state function. Assuming that y is the response variable and the implicit function of the input parameters X, then the response surface $\hat{y}$ is an approximation of this function (Goswami et al. 2016; Hariri-Ardebili et al. 2018; Rajashekhar and Ellingwood 1993) as:

$$y = f(x) = \hat{f}(x) + \varepsilon(x) \rightarrow \hat{y} = \hat{f}(x) \tag{1}$$

where ε (x) is an error in the estimation of the response. A common method for estimating a response surface is the least squares method (LSM). This method is based on the setting of response surface coefficients for a situation which is the best fit for the data obtained from the finite element analysis. The function y is an output quantity which is dependent on n input variables $X_1, \ldots, X_n$. The relation between y and the input variables is expressed as follows:

$$y = f(\beta, x) \tag{2}$$

The function $f(\beta, x)$ depends on the vector $\beta = \left[\beta_1, \beta_2 \ldots . \beta_v\right]^T$. These parameters are determined by least squares estimation method:

$$\beta = \left(X^T X\right)^{-1} X^T Y \tag{3}$$

A first- or second-order polynomial including linear expressions, the interaction effect of variables, and quadratic expressions are used to define the limit state function:

$$\hat{f}(\beta, x) = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} \beta_i x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2 \quad (4)$$

Root Mean Square Error (RMSE) is the criteria used to assess the quality of the estimation of the function using LSM,

$$\text{RMSE} = \sqrt{\sum_{i=1}^{p} (\hat{y}_i - y_i)^2 / p} \quad (5)$$

where p is the total number of data, y is the actual response from the analysis of finite element and $\hat{y}_i$ the predicted response by the equation estimation of RSM.

2.2 Artificial Neural Network (ANN) Method

For probabilistic analysis, Monte Carlo simulation method with a large number of samples is used to evaluate the structural response. Unlike simplicity, this method has a high computational cost due to spending a lot of time. Therefore, the neural network method can be used as an effective tool for solving this problem (Cardoso et al. 2008). As shown in Fig. 1, the structure of a neural network consists of three layers including an input layer, a hidden layer, and an output layer. Each layer has its neurons or nodes and attachment weights. The number of neurons in the hidden layer was fixed by the rule of trial and error. In this way, the number of hidden layers was added one by one to get the minimum error for the outputs (Sheela and Deepa 2013). The results are passed through a nonlinear activation function (transfer function) for each neuron in the network by taking into account the sum of the weighted inputs. These functions are used to transfer values from the hidden layer to the output layer. In this study, the Hyperbolic Tangent sigmoid functions are used based on studies conducted by references (Javidan et al. 2018; Waszczyszyn 1999). The tangent sigmoid function is as follows:

$$y = f(w.x + b) \quad (6)$$

$$f(x) = \frac{2}{1 + e^{-2x}} - 1 \quad (7)$$

For a precise prediction according to the input vector, weights and bias should be well balanced. The process of obtaining these coefficients is called the training process. In this study, a multilayer feed forward network is used because of its ability to estimate functions with high nonlinear degree. This network has also been used effectively in the past studies related to the probabilistic analysis (Chojaczyk et al. 2015; Lagaros et al. 2009). Neural network training methods are performed using uncertainty parameters in the MATLAB (2016) program. The structure response is obtained by the finite element software OpenSees (Mazzoni et al. 2006) with any change in the input parameters. Then, the MATLAB program calls the OpenSees responses to train the neural network. Finally, the Levenberg–Marquardt training algorithm and the number of six hidden neurons are used with trial and error method in order to achieve the best network performance. 70% of the data is used for training network, 15% for validation and 15% for testing data.

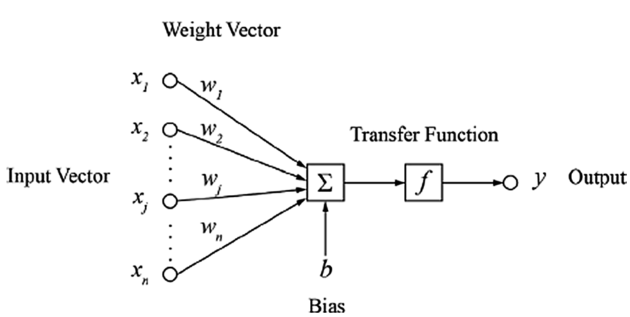


Fig. 1 Configuration of a neural network model (Javidan et al. 2018)

3 Progressive Collapse Analysis

In order to evaluate the progressive collapse of the structures, a push down nonlinear static analysis is carried out under gravity loads in an incremental form with two exterior and interior column removal scenarios. Gravity loads increase in the damaged bays caused by the removal of the column until the rotation of beam above the damaged column reaches the limit states. According to the work done by Conrath et al. (1999), these limit states represent different levels of damage for steel structures under extreme loads, which are shown in Table 1. The values presented in Table 1 correspond to the rotation of the beam in the bays affected by the different damage levels. The combination of gravity load considered for intact bays is (1.2dead + 0.5live) according to DoD guidance (2017). The gravity load applied to the damaged

Table 1 Limit states considered for damaged structures under the extreme loads (in radian)

Element type	Light	Moderate	Severe
Beam	0.05	0.12	0.25

spans should be multiplied by a factor of Ω_N to take into account the dynamic nature of the progressive collapse phenomenon. This factor is calculated as follows:

$$\Omega_N = 1.08 + 0.76/(\theta pra/\theta y + 0.83) \quad (8)$$

where θpra is the plastic rotation and θy is the yield rotation. The calculation process of these parameters can be found in reference (ASCE 2007). After calculating the coefficient Ω_N, the value of 1.33 is considered for all states in deterministic analysis.

4 Model Validation for Analysis of Progressive Collapse

To validate the progressive collapse analysis, the results of the test conducted by Sadek et al. (2010) are used. This experiment was performed on the frame having two bays and the connection of the Reduced Beam Section (RBS), which was subjected to the removal of the middle column. Test setup for specimen with RBS is shown in Fig. 2. In this setup, diagonal braces with cross section of w14 × 109 were used to simulate the effects of bracing created by upper floors. A hydraulic ram with capacity of 2669 kN was used to apply the vertical load to the middle column. Two cross

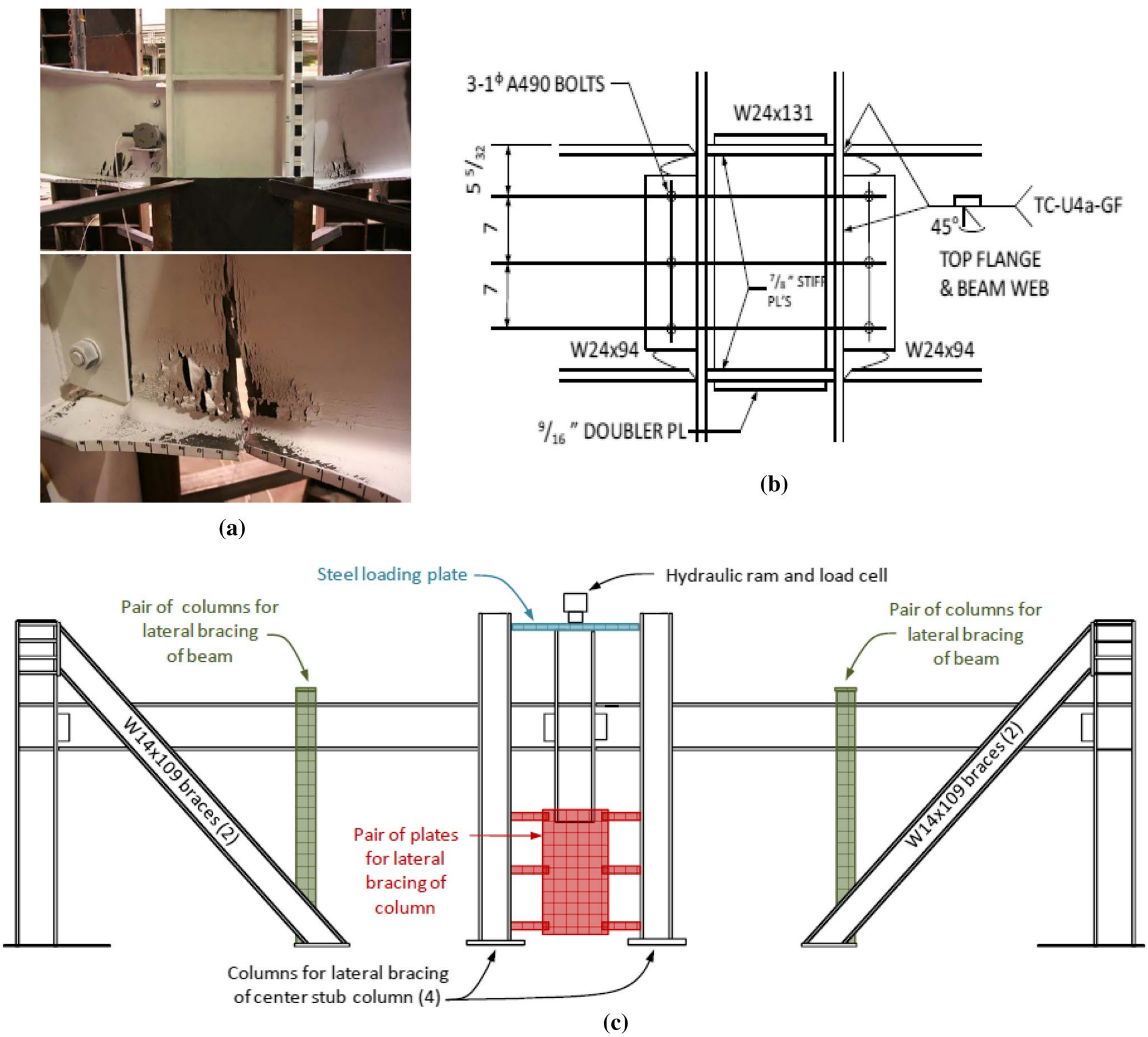


Fig. 2 Details of test setup for specimen (Sadek et al. 2010): **a** failure mode in specimen; **b** details of RBS connection; **c** elevation view

sections of w24 × 94 and w24 × 131 were used for the beams and the columns, respectively. The properties of steel materials are presented in Table 2.

The analysis of the model was performed in OpenSees software. An elastic perfectly plastic model was used based on Table 2. The panel zones were also modeled using a diagonal spring. Shear behavior of panel zone was determined by the stiffness and strength formulated in the work of Khandelwal et al. (2008). The results obtained from the laboratory test for the axial force induced in the beams are compared with results of the model analysis in Fig. 3. As can be seen from Fig. 3, the axial force developed in the beam above the removed column is 2450 kN. These values show the effect of the catenary action in the beams which indicate their resistance against large deformations. Furthermore, the results of finite element analysis for proposed model are fairly consistent with the experimental results.

4.1 Details of Modeling Structure and Material

The modeling examined in this study is an 8-story steel building, so that its progressive failure process was previously investigated by Jin and El-Tawil (2005). This model has four bays in both of the longitudinal and transverse directions with the same lengths of 9.14 m. The height of the first floor is 4.57 m, and the heights of the other floors are equal to 3.66 m. The peripheral special moment-resisting frames are considered as the lateral load-resisting system, and the interior frames are regarded as the gravity load-resisting system. Plan and elevation views of the structural model are shown in Fig. 4. The dead loads applied to floors and roof are, respectively, 5 kN/m^2 and 3 kN/m^2, while the corresponding live loads are 2.4 kN/m^2 and 0.96 kN/m^2, respectively. This structural model was designed as a standard office building located in an area close to Los Angeles, based on the soil type used during design of the structure with site class C, $S_S = 2.48$ g, and $S_1 = 1.02$ g. The dimensions of the cross section used for all members of the structure are given in Table 3. Modulus of elasticity and yield strength for beams and columns are equal to 2×10^5 MPa and 288 MPa, respectively. Nonlinear analysis is performed using an open source software such as OpenSees (Mazzoni et al. 2006) by considering a special moment-resisting frame periphery, as shown in Fig. 4. For modeling the beams and the columns, the nonlinear beam column element with 5 integral points is used. Nonlinear materials steel 01 for columns and reinforcing steel for beams are considered according to the work done by Kim and An (2009). The "Corotational" geometric transformation is used to consider the effect of catenary action in the beams after removing the columns. For columns, the "PDelta" geometric transformation is also used.

Table 2 Properties of steel materials used in test

Component	Yield stress (MPa)	Ultimate stress (MPa)	Yield strain	Ultimate strain
Beam	455	554	0.0024	0.143
Column	378	494	0.0018	0.189

5 Collapse Criterion for Instability of the Column

The stability of columns in the structures under progressive failure can be evaluated by examining the failure of columns adjacent to the removed column. Two scenarios for column removal were considered, which are included the removal of the exterior column C_{11} and the interior column C_{12} as shown by the cross sign on the columns in Fig. 4. Instability

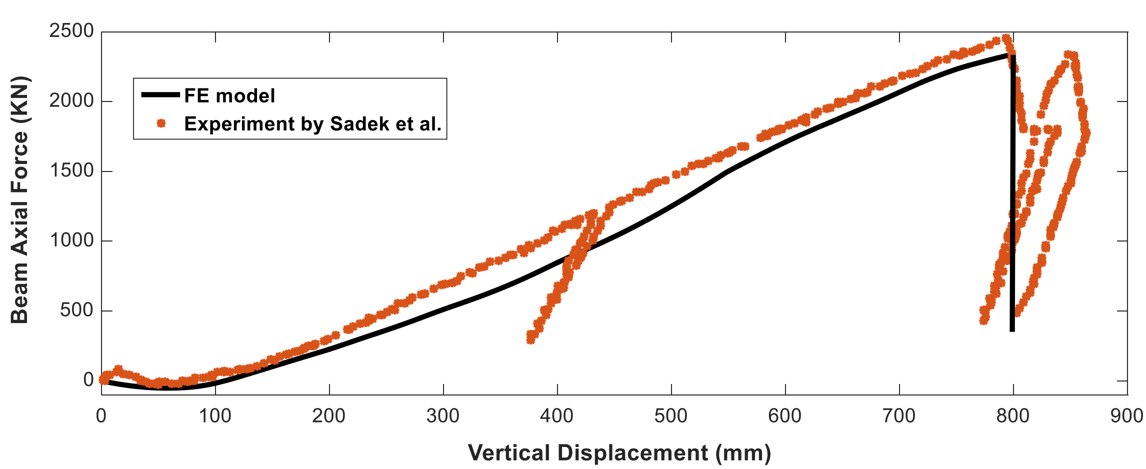


Fig. 3 Comparison of the axial forces induced in the beam obtained from the analytical and the experimental results

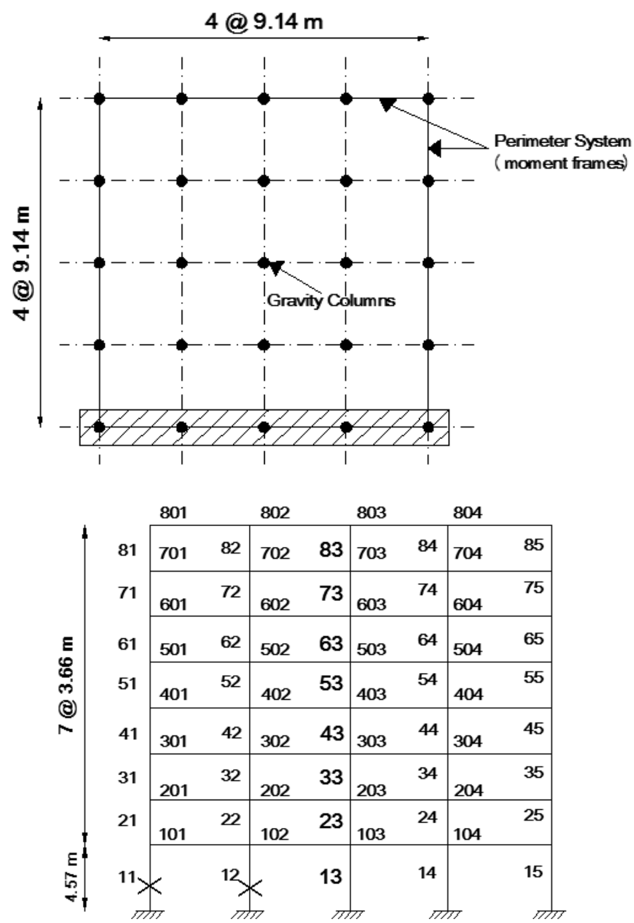


Fig. 4 Plan and elevation view of the structural model with scenarios of the column removal

Table 3 Sections of the structural members

Floor	Beam	External column	Internal column
Roof	W18×60	W14×99	W14×132
7	W21×83	W14×99	W14×132
6	W21×93	W14×109	W14×176
5	W27×102	W14×109	W14×211
4	W30×108	W14×132	W14×233
3	W30×116	W14×145	W14×257
2	W30×116	W14×159	W14×257
1	W30×124	W14×283	W14×342

mechanism occurs when the axial force in the columns adjacent to the damaged column reaches its critical load. The values of this critical load are presented in Table 4 according to the work done by Pantidis and Gerasimidis (2017). After removing the column, additional forces are imposed on the surrounding columns due to the redistribution of forces to maintain system stability, which may cause non-elastic buckling in adjacent columns. In this case, the axial force

Table 4 Critical Load in the columns

Type of column	Condition	Critical load
Slender	$P_{\text{Euler}} < A_c \times f_y$	$P_{\text{Euler}} = \frac{\pi^2 \times E \times I}{(K \times H^2)}$
Intermediate	$P_{\text{Euler}} > A_c \times f_y$	$A_c \times f_y$
Stocky	$P_{\text{Euler}} > A_c \times f_u$	$A_c \times f_u$

of the columns reaches the yielding capacity (P_y) which is equal to $A \times f_y$ where A is the cross section of the column and f_y is the yield strength (Gerasimidis et al. 2015). Buckling occurs in the column when a horizontal displacement happens in the middle of column. In order to reveal the buckling mode of columns needs to consider the imperfections in modeling. As suggested by Pantidis and Gerasimidis (2017), these values are assumed to be 0.001 times the vertical loads which are entered in a horizontal load on each floor and do not affect the structural responses.

In this study, a deterministic analysis is performed on the structure under three damage states including light, moderate, and severe with two scenarios for removal of the columns in the interior and the exterior situations. In these analysis, the effect of the catenary action is considered in the beams. Then, the ratio of the axial force demand to the inelastic buckling capacity P/P_y corresponding to the columns adjacent to the damaged column is extracted to determine the buckling mode in each limit state. The buckling occurs when the ratio P/P_y is greater than the value of one, which means that the axial force demand exceeds the value of buckling capacity of each member.

Considering the catenary action leads to the development axial force in the beams, which increases the resistance of the beams against the removal of the column. Figure 5 shows the axial force obtained from the push down analysis for the beam above the removed column with the number 101 in two removal scenarios. As it can be seen from Fig. 5, the axial force is almost equal to zero when the catenary action is ignored.

The axial force increases in the beam until the limit state is close to severe and then decreases when the beam loses its resistance against the removal of the column. The increase in axial force of the beam is greater in the case of removing the middle column than the corner column, which makes it more resistant against the deformation. The force generated in the beam decreases faster by removing the exterior column. Therefore, resistance of the structure decreases by removing the exterior column faster compared to removing the interior column. Increasing the axial force in the beams will increase the strength of the structure against the progressive failure and the failure will occur later. Therefore, development of the axial force in the beams affects other structural responses.

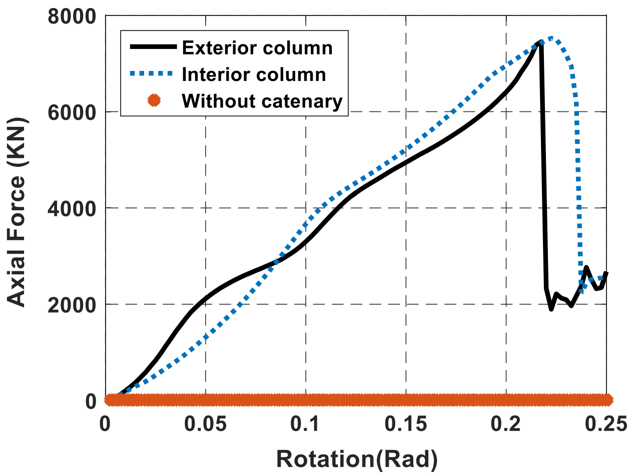


Fig. 5 Comparison of the axial forces created in the beam after the removal of the exterior and interior columns with and without considering catenary action

The maximum value of the ratio P/P_y in the columns adjacent to the removed columns is shown in Fig. 6 for three limit states with considering the catenary action. As shown in Figs. 6 (a) and 6 (b), buckling occurs in two columns C_{11} and C_{13} after removing the interior column and in column C_{12} after removing the exterior column in severe limit state. Results obtained for the ratio P/P_y are approximately the same for two columns C_{13} and C_{14} and have small distance to value of 1 in removing the exterior and the interior column, respectively. From the results, it can be seen that more columns buckle in the severe limit state than two other limit states in a special moment frame. While in the light limit state, none of the columns buckle. Since these columns may behave differently under uncertainty conditions, they are considered as a case study in probabilistic analysis for a more accurate examination in severe limit state. Therefore, the effect of each uncertainty parameter on the ratio P/P_y for these columns is quantified.

6 Probabilistic Analysis of the Structures Under the Progressive Collapse

Structural responses may be affected by the uncertainty parameters when the structure undergoes a progressive collapse. Kirçil and Polat (2006) have shown that the capacity of structures under progressive failure depends on the variability of the design variables. Therefore, these uncertainties can be taken into account in the specification of the materials and loadings which are ignored in the existing regulations.

In this study, the uncertainty parameters considered are the elastic modulus, yield strength of the structural members, live load, and dead load. The correlation coefficient is assumed to be 0.2 between the yield strength and the modulus of elasticity (Park and Kim 2010). Table 5 shows the mean, the standard deviation, and the distribution function of each random variable (Bartlett et al. 2003; Ellingwood 1980).

Performing a probabilistic analysis requires a limit state function g (x). This function can be determined based on the ratio P/P_y of the columns adjacent to the removed column as an implicit function:

$$g(X) = 1 - P/P_y \tag{9}$$

where X is the vector of random variables that change at each stage of the analysis. The failure mode occurs when g (x) ≤ 0. RSM method can be used to extract an explicit

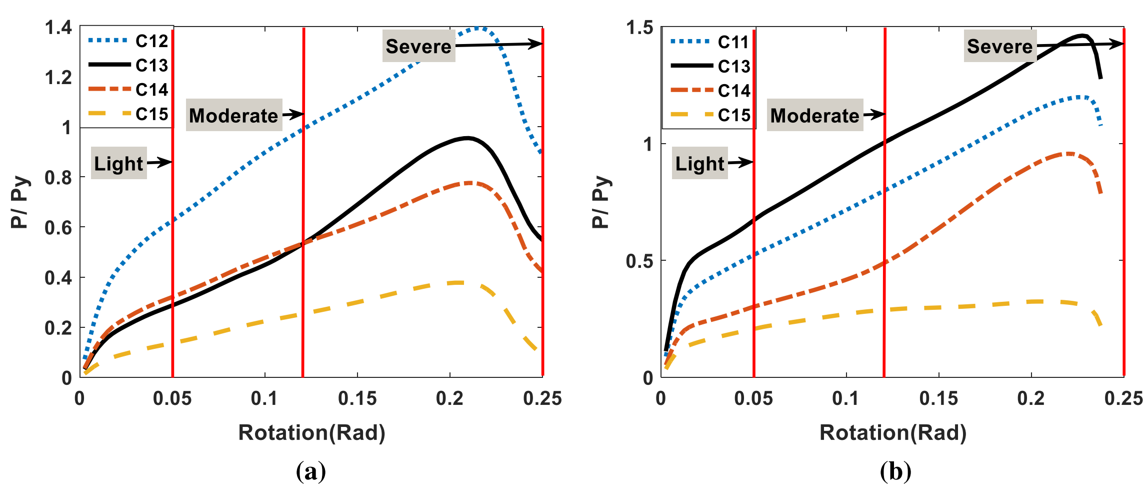


Fig. 6 The ratio P/P_y for columns with considering catenary action: **a** the removal of the exterior column C_{11}; **b** the removal of the interior column C_{12}

Table 5 Statistical properties of uncertainty parameters

Variables	Mean	Coefficient of variation	Probability distribution	References
Yield strength (f_y)	$1.10F_{yn}$	0.06	Normal	(Bartlett et al. 2003)
Elastic modulus (E)	$0.993E_n$	0.034	Normal	(Bartlett et al. 2003)
Dead load (DL)	$1.05D_n$	0.1	Normal	(Ellingwood 1980)
Live load (LL)	L_n	0.25	Normal	(Ellingwood 1980)

function for the engineering demand parameter (EDP) which is the axial force here. To estimate the limit state function by RSM, five levels are considered for each of the random variables, which include mean, mean ± 1 standard deviation and mean ± 2 standard deviation. In total, 5^4 finite element analysis are needed to perform for 4 random variables. Finally, 625 number of responses are extracted. Then, they can be utilized to estimate the EDP based on the random variables using a quadratic polynomial model as Eq. (4) as follows:

$$\widehat{EDP} = \beta_0 + \beta_1 E + \beta_2 fy + \beta_3 DL + \beta_4 LL + \beta_5 Efy + \beta_6 EDL + \beta_7 ELL + \beta_8 DLfy + \beta_9 LLfy + \beta_{10} DLLL + \beta_{11} E^2 + \beta_{12} fy^2 + \beta_{13} DL^2 + \beta_{14} LL^2 \tag{10}$$

where $\widehat{EDP}$ is the estimation of the axial force demand for the columns. The response values can also be considered as target data for the proposed neural network. Therefore, a vector of 1 × 625 as target data and a vector of 4 × 625 are used as input data for training and testing of the neural network.

The probability of failure is estimated using the Monte Carlo analysis based on both the above-mentioned methods as follows:

$$P_f = \frac{1}{N} \sum_{i=1}^{N} I(X_i) \tag{11}$$

N is the number of samples and I is the failure index, which is displayed with the values of 0 and 1 as

$$I(X) = \begin{cases} 1 & g(x) \le 0 \\ 0 & g(x) > 0 \end{cases} \tag{12}$$

7 Performance of the Neural Network in Estimating Results

An example of performance of the ANN for predicting the structural response of column C_{13} to achieve the minimum mean squared error for both the column removal scenarios can be seen in Fig. 7. As it can be seen, the trained network can predict accurate results with training, validation, and testing samples with a very small error. Best validation performance is achieved with MSE 2.1724×10^{-5} at epoch 7 in interior column removal. Also, best validation performance is 4.9068×10^{-9} at epoch 75 for the case of removing exterior ones. Therefore, this well-trained network can be used to perform probability analysis.

The results obtained from the nonlinear analysis using finite element analysis and the estimation of the ANN in validation stage of data have been selected to capture the ratio P/P_y of the columns adjacent to the removed column

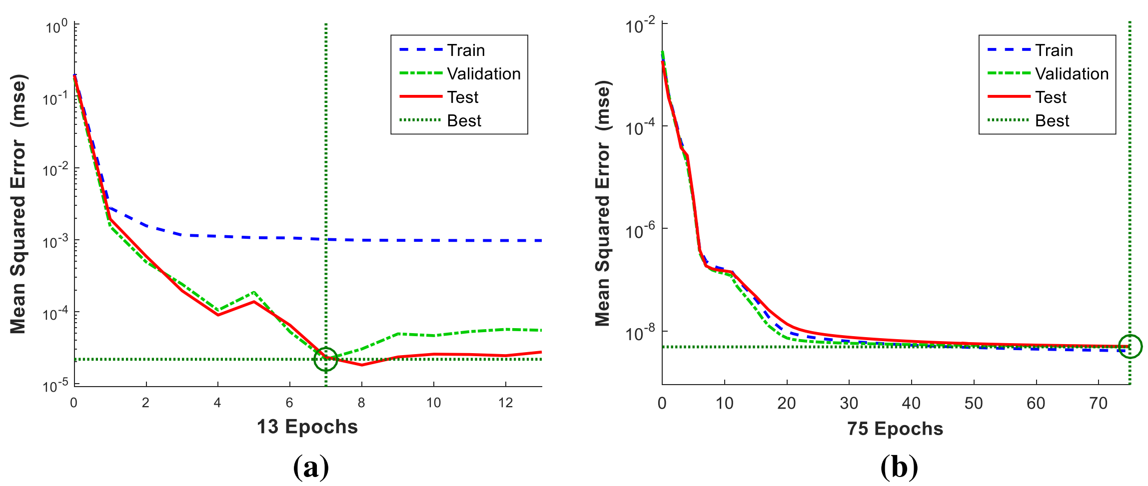


Fig. 7 Reduction in MSE in the training of the network: **a** the removal of the interior column C_{12}; **b** the removal of the exterior column C_{11}

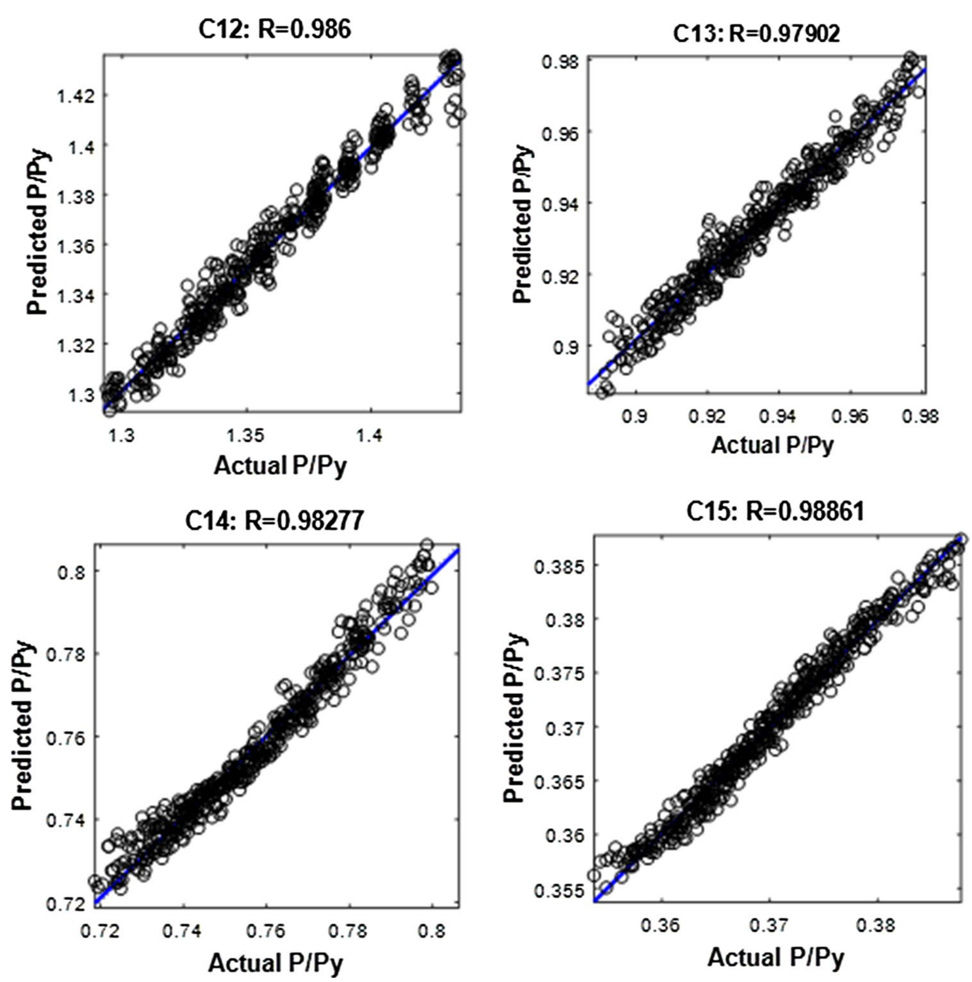
in Figs. 8 and 9. These results indicate that there is a good correlation between the results of the both analysis. Coefficient of correlation R is approximately equal to 1. As a result, there is a very good fit between results obtained from the finite element analysis and the results predicted by ANN.

8 Results of Probabilistic Analysis for Columns Adjacent to the Damaged Column

In this study, after examining the quality of the trained network and RSM method based on the criteria mentioned in the previous sections, the probabilistic analysis is performed using Monte Carlo simulation. The values of ratio P/P_y are calculated for columns using the Latin hypercube sampling for 10^6 number of samples in both the removal scenarios by the ANN and RSM methods. The numerical results obtained from the probabilistic analysis are shown in Figs. 10 and 11 in form of probability distributions. Figure 10 shows the probability density of each of the values obtained for the ratio P/P_y of columns when an exterior column is removed. As shown in Fig. 10, all values of this ratio for column C_{12} are concentrated in range 1.3–1.4 which is greater than the value 1, while the concentration of these values for the other columns is in the range of less than the value of one. Therefore, column C_{12} certainly buckles. The results obtained for the probability density are a very good match in both RSM and ANN methods. As shown in Fig. 11, the ratios P/P_y for columns C_{11} and C_{13} are concentrated in the range of 1.1–1.3 and 1.3–1.5 in the case of interior column removal, respectively. These values are in the range of less than the value of one for columns C_{14} and C_{15}. Therefore, columns C_{11} and C_{13} certainly buckle.

The probability that the values obtained for P/P_y are less than a certain value can be displayed in the curves that known as cumulative distribution functions (CDFs) as shown in Fig. 12. These curves for each column are extracted by fitting these ratios with a lognormal distribution function for the both RSM and ANN methods. Therefore, it is necessary to examine the probability of values that are less than the value of one due to the buckling does not occur in this situation for columns around the damaged column. As can be seen, there is a good match between the

Fig. 8 Comparison of the actual values calculated from the finite element analysis and the values predicted by ANN for the ratio P/P_y of columns in removal of the exterior column C_{11}



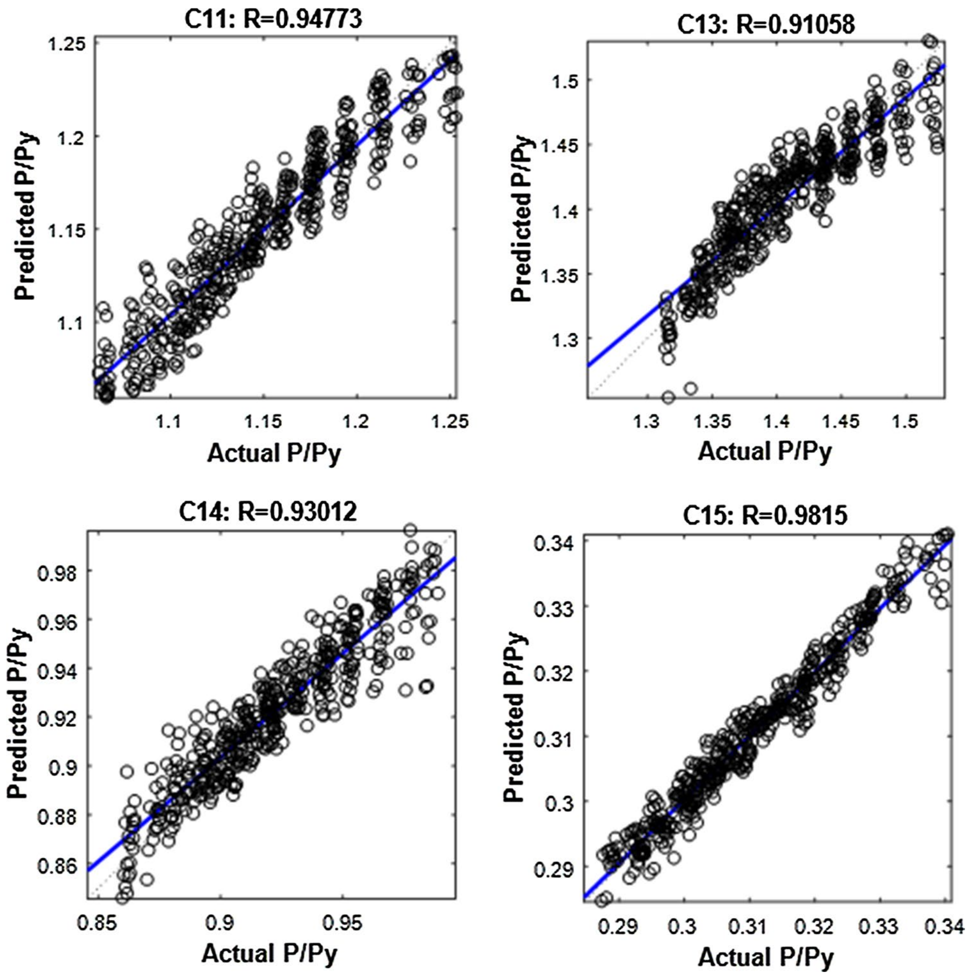


Fig. 9 Comparison of the actual values calculated from the finite element analysis and the values predicted by ANN for the ratio P/P_y of columns in removal of the interior column C_{12}

results of CDFs from the both RSM and ANN methods. The maximum values of ratio P/P_y which is corresponding to probability 100% extracted from the CDF diagrams that shown in Fig. 13. It can be seen that in the case of interior column removal, these values are less than the value of one only for column C_{15} and also for columns C_{14} and C_{15} in case of exterior column removal. Therefore, buckling in these columns does not occur under both the removal scenarios.

The exact values of the probability of the buckling of the columns around the damaged column obtained from the Monte Carlo analysis that are summarized in Tables 6 and 7. These results obtained for the neural network and the response surface methods for both the column removal situations. Also, the accuracy of the estimation of the probability failure using these two methods is measured with the values of MSE.

As shown in these Tables, MSE obtained for the neural network method is estimated more accurately than the response surface method. However, there are very little difference in probability of failure derived from two ways in some columns. This difference can be due to the high nonlinearity of the structural responses in the severe limit state. Therefore, the response surface method may estimate the limit state function with less accuracy in some conditions compared with ANN. According to Tables 6 and 7, the probability of failure has the same results for both methods in both removal scenarios.

By comparing the results of Tables 6 and 7, it can be found that the probability of buckling is very high for columns of C_{11} and C_{13} in removal of the interior column and is also very high for column C_{12} in case of the exterior column removal. Therefore, the results of the probabilistic analysis are very well suited to the results of deterministic analysis that shown in Fig. 6. In case of the interior column removal, the buckling probability of the columns C_{14} and C_{15} is very small. In case of the exterior column removal, the buckling probability of the columns C_{13}, C_{14} and C_{15} is also very small. These results are highly consistent with the results obtained in previous sections.

9 Sensitivity Analysis

In sensitivity analysis, the effect and importance of the variability of each of the random variables on the response of the structure are determined. At first, the structure response is

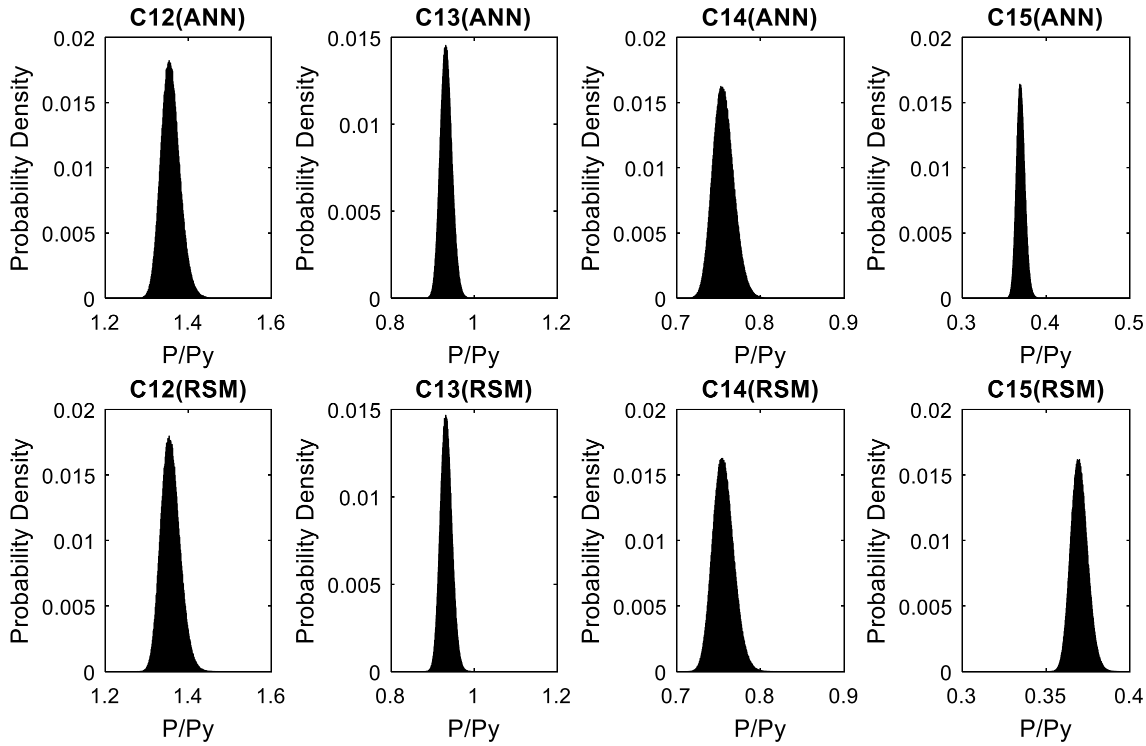


Fig. 10 Probability distribution for ratio P/P_y of columns using RSM and ANN methods under exterior column removal

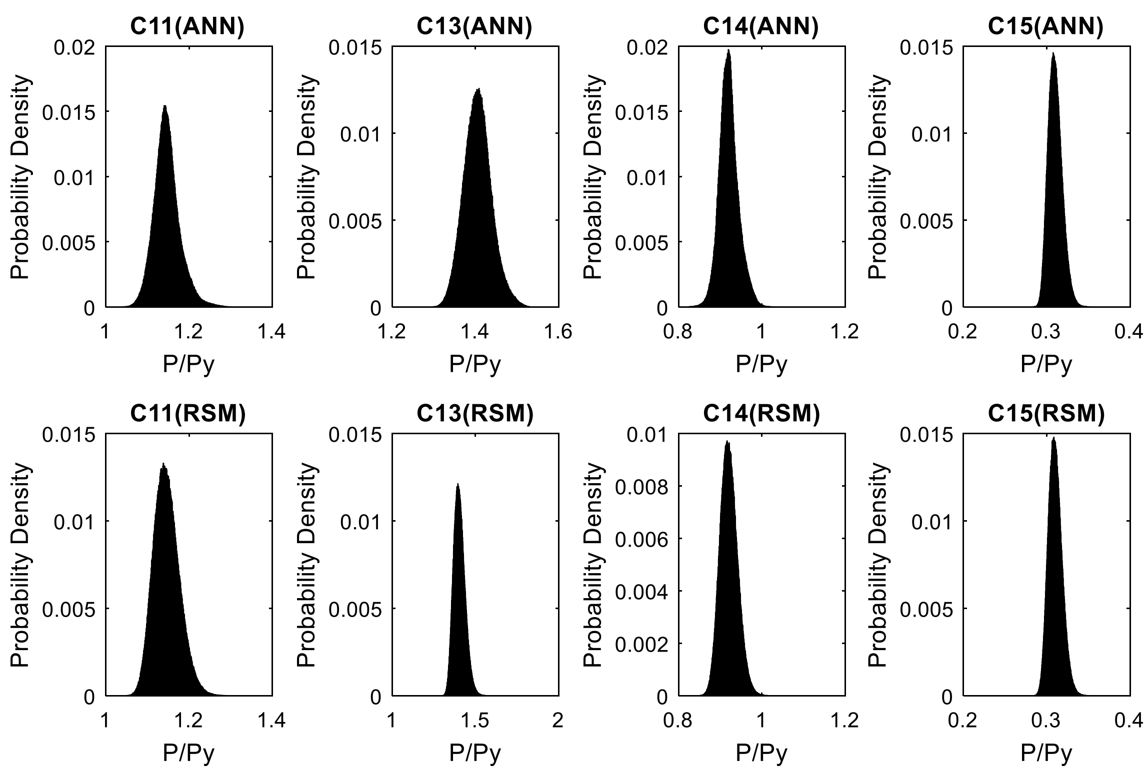


Fig. 11 Probability distribution for ratio P/P_y of columns using RSM and ANN methods under interior column removal

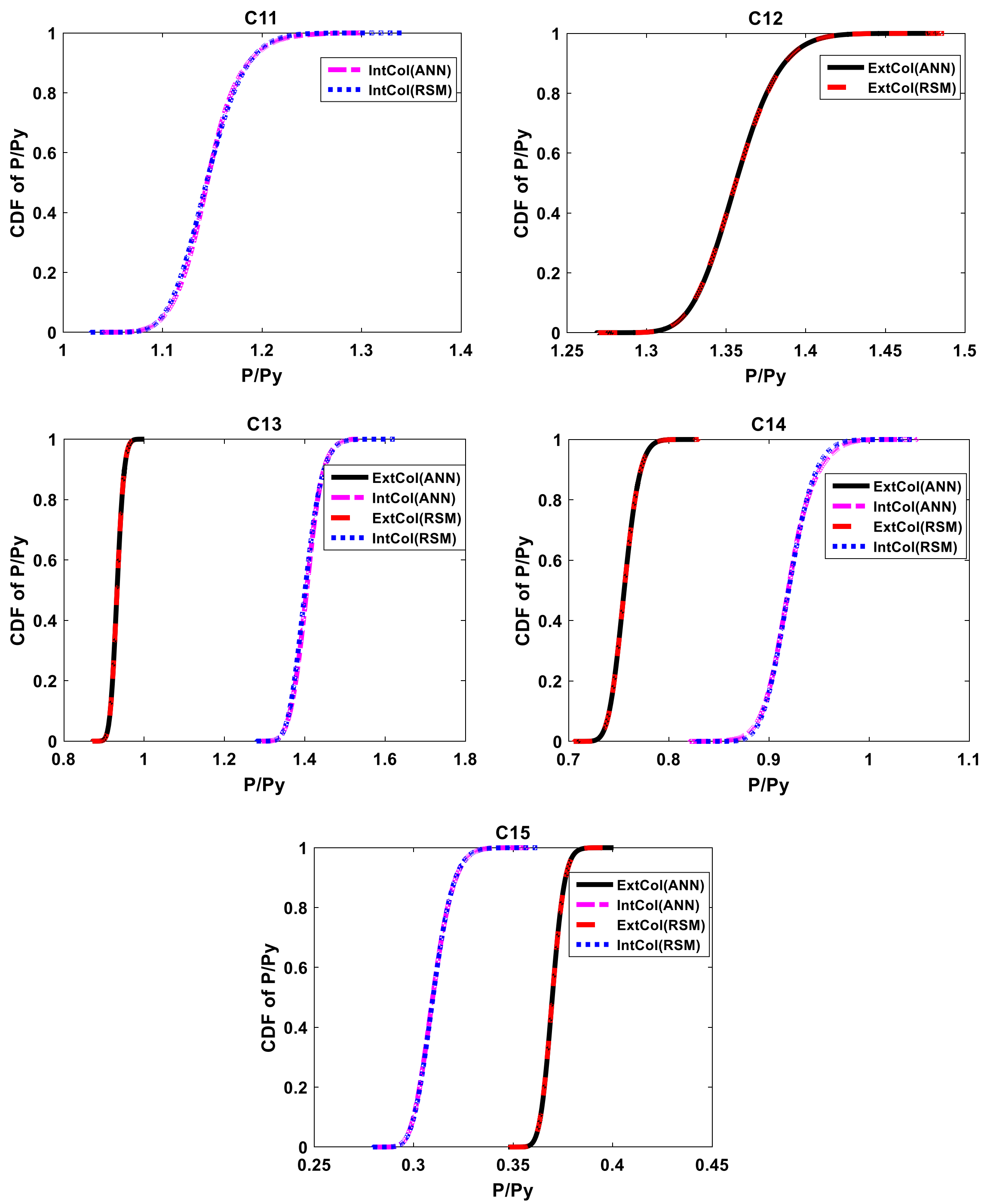


Fig. 12 Comparison of the cumulative distribution functions of the ratio P/P_y of columns in both column removal scenarios using RSM and ANN methods

obtained for the mean value of each input parameter. Then, this mean of the response obtained is selected as the base value. The least and most responses of the structure are determined by 10^6 number of samples similar to the Monte Carlo analysis according to the distribution function of each variable, while the values of other variables do not change and their response is based on their mean value.

As the results obtained in the previous section, the maximum value of the ratio P/P_y occurred in columns C_{12} and C_{13} under removal of exterior and interior column scenarios, respectively. Therefore, these two columns are the most vulnerable columns in the column removal process and the sensitivity of the response of these columns is considered relative to the variability of each random

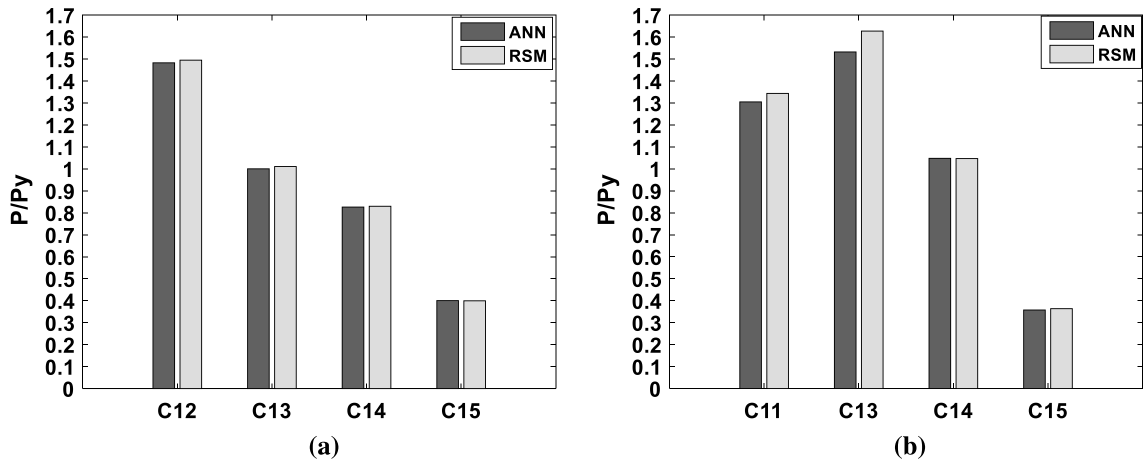


Fig. 13 Maximum ratio P/P_y of columns with 10^6 samples: **a** the removal of the exterior column C_{11}; **b** the removal of the interior column C_{12}

Table 6 Failure probability of the columns adjacent to the exterior column removal

Column number	RSM method		ANN method	
	Failure probability (P_f)	Mean square error (MSE)	Failure probability (P_f)	Mean square error (MSE)
$\mathbf{C}_{12}$	0.994	0.0008	0.989	3.023×10^{-8}
$\mathbf{C}_{13}$	7.7×10^{-6}	0.0001	3×10^{-7}	4.90681×10^{-9}
$\mathbf{C}_{14}$	2.2×10^{-6}	0.0002	1.1×10^{-7}	1.34554×10^{-8}
$\mathbf{C}_{15}$	0.5×10^{-6}	0.0000	0.1×10^{-7}	2.44816×10^{-9}

Table 7 Failure probability of the columns adjacent to the interior column removal

Column number	RSM method		ANN method	
	Failure probability (P_f)	Mean square error (MSE)	Failure probability (P_f)	Mean square error (MSE)
$\mathbf{C}_{11}$	0.995	0.0025	0.991	3.7368×10^{-5}
$\mathbf{C}_{13}$	0.989	0.0038	0.984	2.1724×10^{-5}
$\mathbf{C}_{14}$	4.52×10^{-4}	0.0009	0.001	1.2799×10^{-4}
$\mathbf{C}_{15}$	0.02×10^{-4}	0.0002	0.3×10^{-3}	9.99452×10^{-7}

variables. Variability of the structural response to any of the uncertainty parameters is shown in Fig. 14, known as Tornado diagram. The contribution of each random variable on variability of the structural response is determined by swing, which is the difference between the highest and the lowest response. It can also be seen that among these four uncertainty parameters, most swing is related to the yield strength, which has the greatest effect on the ratio P/P_y of column, and the second ones is corresponding to the modulus of elasticity. Changes in dead load and live load do not have a significant effect on the variability of this ratio in the columns.

10 Conclusions

In this study, the behavior of a special steel moment-resisting frame structure under progressive collapse was investigated using the probabilistic analysis methods. Monte Carlo simulation was used to estimate the failure probability of the structure. Since the limit state function of the structural responses to the uncertainty parameters is an implicit function, it takes a lot of computational time to perform the Monte Carlo analysis using these functions. Therefore, the response surface method was used

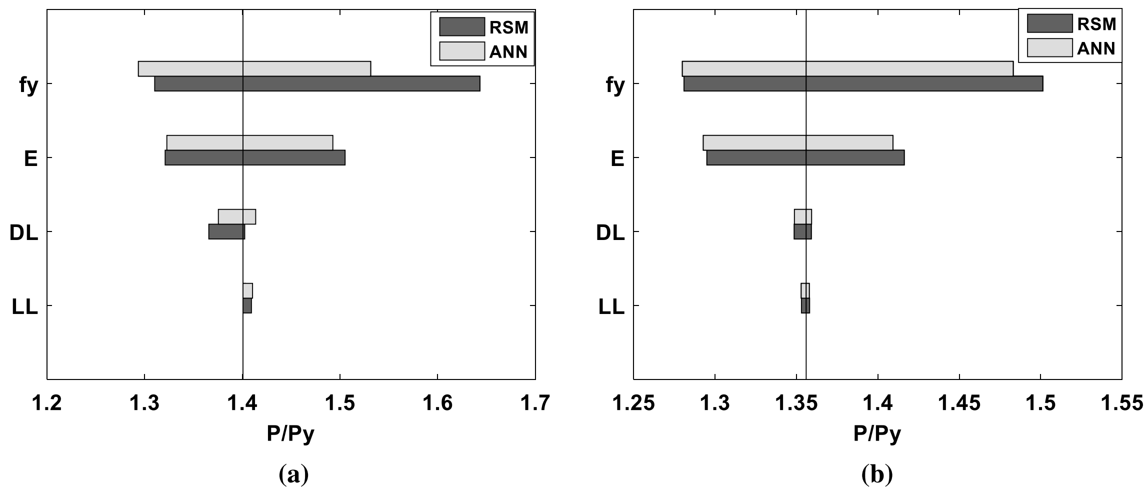


Fig. 14 Tornado diagram derived from the sensitivity analysis based on ANN and RSM methods: **a** column C_{13} in the removal of the interior column; **b** column C_{12} in the removal of the exterior column

to estimate an explicit function that relates structural responses to random variables. The mean squared error (MSE) criterion was used to determine the accuracy of the estimation of the limit state function using the response surface method. Neural network method was also used for verifying the response surface method. To demonstrate the applicability of these methods, an 8-story steel special moment frame structure was examined under two scenarios of the column removal. Then, the behavior of the columns adjacent to the damaged column was studied by constructing a limit state function, which was based on the ratio of the demand axial force of each column to the non-elastic buckling capacity (P/P_y) using the probabilistic analysis with the RSM and the ANN methods. The conclusion of this study can be summarized in the following states:

- The number of finite element analyzes was reduced by using Monte Carlo simulations based on RSM and ANN methods that found limit state functions based on random variables. Also, the results of probabilistic analysis with 10^6 number of samples have very little error in both methods.
- Results obtained from the RSM and the ANN methods were in good agreement especially when the exterior column was removed. In the case of removing the exterior column, the mean squared error (MSE) in both methods was less than the interior column removal scenario. Comparison of the responses obtained from both the RSM and the ANN showed that the ANN method can estimate the structural responses with less error.
- The results of cumulative distribution functions (CDFs) showed that columns C_{11} and C_{13} in the case of interior column removal and column C_{12} in case of exterior column have a very high probability of buckling. On the other hand, columns C_{14} and C_{15} in removing the exterior column and column C_{15} under the scenario of the interior column removal have a very low probability of buckling.
- According to the sensitivity results, the ratios of P/P_y were more affected by the uncertainty parameters of the yield strength and the modulus of elasticity, respectively. Dead load and live load have little impact on the variability of this ratio in columns.

The results of the sensitivity analysis can be used to retrofit the structures affected by the progressive failure with controlling the parameters affecting the vulnerability of the columns adjacent to the damaged column. For a decision-maker to rehabilitate damage structures, ignoring uncertainties is highly unconservative. Therefore, the effect of uncertainties on structural responses should be determined by probabilistic analysis. Also, the most effective uncertainty parameter is determined using sensitivity analysis. One may want to retrofit the structure under progressive failure for a certain level of performance. To control the vertical displacement response above, the removed column can be used the results of sensitivity analysis and can be applied materials for retrofitting by considering the most effective uncertainty parameters on the structural response.

There are several limitations in the present study. Firstly, we performed numerical modeling using 2D frames to simplify and facilitate the use of probabilistic analysis to solve problems such as convergence and computational time. Secondly, in the probabilistic analysis using Monte Carlo simulation need to be considered a large number of samples to achieve the probability of failure with high accuracy. We performed this analysis with 10^6 samples that finite element analysis could not estimate the structural responses due to time consuming and convergence problems. To overcome the problem of insufficient samples, we used the response surface method to estimate an equation for structural responses based on random variables by accepting a small MSE. Also, ANN method was used to validate the RSM. Due to the time-consuming nature of the probabilistic analyzes, we only investigated the buckling failure mode in the columns while other failure modes such as the failure of the connections and yielding of the bending members such as beams were not considered for structures under extreme loads, Therefore, further research is needed about the issues mentioned in the future.

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Data Availability All data, models, and code generated or used during the study appear in the submitted article.

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